speculum, then that of the largest opening of the diaphragm will be $D - 2av = D - \frac{4a}{3} \tan \alpha$ for the limiting value of α.

If now we assume $\alpha = 2°$ we shall find the diameter of the diaphragm about $43\frac{1}{4}$ inches. The image for the field will be 25·14 inches in diameter, and probably the carriers will require to have a space of 27 inches in diameter allowed for them. This would leave only 28 per cent. of the surface available to form the image of any object; excluding all the central part.

On the Atmospheric Transmission of Visual and Photographically Active Light. By Captain Abney, R.E., F.R.S.

In his publication, 'Researches on Solar Heat,' which appears in the Professional Papers of the Signal Service, Professor S. P. Langley has been somewhat hard on those astronomers who have made a speciality of observing star-magnitudes. In the report on the transmissibility of our atmosphere for light, in which is embodied the observations made by his party at Mount Whitney, and which appears in the above-named volume, he makes the following remarks: 'For it may be observed in general terms that since the rays with large coefficients are represented by diminishing geometric progression, whose common ratio is near unity, these rays will persist whilst others with small coefficients are very nearly extinguished; and something like this was shown by Biot at the time when Melloni's first observations on the transmission of heat through successive strata attracted attention. But what we desire now further to point out is, that according as the difference of these coefficients of transmission for the different portions of the light of the same star is greater, so will the error of the result in treating them as equal be larger—a consequence so obvious that it is only necessary to make the statement in order to have its truth recognised.

'Since it has now been demonstrated that the formula ordinarily employed leads to too small results, it might properly be left to those who still employ it to show that their error is negligible; but this has never been done. There is possibly an impression that if there were any considerable error its results would become apparent in such numerous observations as have been made all over the world in stellar photometry during this century. But it is, in my opinion, a fallacy to think so; and I believe, as I have elsewhere tried to show, that the error might be enormous; that the actual absorption *might* be twice what it is customarily taken, or 40 per cent. instead of 20 per cent., without the error being detected by such observations as are now made.'

In a paper which I have recently communicated to the Royal Society, on the transmission of sunlight through our atmosphere, I

have described in detail observations which I have carried out during the past year on this subject, using the visible solar spectrum for the particular object I had in view. My wish was to ascertain the loss of luminosity of each visible ray after transmission through varying atmospheric thicknesses. The definite conclusion which I came to was that, as a rule, the loss of light followed Lord Rayleigh's law for scattering by small particles—

$$I' = I\epsilon^{-kx\lambda^{-4}},$$

where I′ and I are the transmitted and original intensities, x the thickness of air, and λ the wave-length. These observations which I made it is needless to describe again in this note; suffice it to say they were made by means of the colour photometer described by General Festing and myself last year in the Bakerian lecture. My standard luminosity curve was one taken at 8,500 feet altitude in the Alps, and, as I have already said, nearly every curve of luminosity through other air thicknesses obeyed the above law. Now, having got this result, it was easy to construct curves of luminosity of the spectrum for 1, 2, 3, . . . &c. air thickness in which the ordinates were the relative intensities of the rays transmitted. The areas of such curves when constructed could very easily be found, and such areas would represent the total luminosity of white light coming from the Sun though with varying thicknesses. It would then be easy to see what variation from the law usually employed in stellar observations would occur, and which, according to Langley, would be serious. The law in question is that—

$$I' = I\alpha^{\sec\theta},$$

I′ and I being the transmitted and original intensities and α the coefficient of transmission, θ being the zenith distance, the secant of which, within certain limits, is a measure of the air thickness. I found that the minimum value of k in Lord Rayleigh's formula was ·0013, when the value of λ^{-4} for $\lambda=6000$ was taken as 78·3. In this case the areas of the curves representing 0, 1, 2, 3, 4, and 5 atmospheres were represented as 761, 662, 577, 504, 439, and 385 respectively. It may be supposed that an observer when observing sunlight integrally would have obtained the same values, and then attempted to adapt the formula just given to it. Probably he would have used 'least squares,' and arrived at a result, but it would hardly have been necessary to do so. I had first omitted to calculate the area of the curve of the spectrum through 5 atmospheres, and I simply took 0 and 4 atmospheres as lying on the curve by the formula—

$$I' = I\epsilon^{-\mu x},$$

where μ is the coefficient of absorption, and in which $\epsilon^{-\mu}=\alpha$ in the above formula. From this I found that $\mu=·1378$, and this

gave for 1, 2, and 3 atmospheres 664, 578, and 504, the results from the spectrum being 662, 577, and 504. Calculating next the absorption through 5 atmospheres, I found it to be 383 instead of 385. The coefficient of transmission in the usual formula from the above is $a = \cdot 862$, a somewhat high value; but it must be recollected it was my maximum value. Taking a larger value of k, viz. $k = \text{0019}$, the areas for 0, 1, 2, 3, and 4 atmospheres were 917, 755, 623, 513, 418. Taking the first and last as lying on the logarithmic curve, the values obtained were 917, 755, 626, 508, 418, a very close approximation, μ being ·19708, and the coefficient of transmission being ·822. I may mention that with any value for k that I tried the same result held good. It thus appears that the logarithmic formula is more than a close approximation to the truth with the values arrived at from my observations.

Professor Langley's values and my own for the transmissibility of the different visible rays differ somewhat, so it occurred to me that it would be right to take his own values for transmissibility, and treat them in the same manner. Taking my Riffel observations of the luminosity of the spectrum as correct, I calculated from them the value of the areas of the curves of luminosity, using Langley's own coefficients of absorption. They were as follows, 1, 2, 3, 4, and 5 atmospheres, 657, 392, 235, 142, and 52. Taking 1 and 4 atmospheres as being on the logarithmic curve, the values derived from the formula were 657, 394, 237, 142, and 51, μ being ·5106 and a being ·602; a value which is extremely low.

This shows that still the agreement is complete between them. It must be borne in mind that in observations such as these, which are for settling stellar magnitudes, there is no desire to know the intensity of light outside our atmosphere. All that concerns the astronomer is to compare light coming from a star at the zenith with that coming from it at a reasonable zenith distance; 5 atmospheres is represented by a Z.D. of about 78° 30′, and it is unlikely that anyone would set much value on observations made at a greater Z.D. than that. Hence for the comparison of star magnitudes the usual formula may safely be adopted. A much more serious matter is the absorption coefficient used, and we find that it varies between ·791 and ·843, according to previous observations, whilst I have found my maximum at sea level to be ·869, though my mean is not far from the ·825, which value has been adopted by some astronomers.

The Z. distances of two observed stars may vary very greatly. This discrepancy of course would alter the calculated magnitudes very considerably, but if the difference be small the calculated magnitude will not be far out, even if the absorption coefficient be a little out from the truth. I may remark that in taking his stellar magnitudes, Professor Pritchard seems to have taken every care as to this, his comparison star being *Polaris*, and the star observed being about the same zenith distance.

Having arrived at the conclusion regarding visual observation, it struck me that absorption of the photographic spectrum ought to follow the same course. Now in the 'Proceedings of the Royal Society' I have given a very careful measurement of the photographic sensitiveness to various rays of light of bromo-iodide of silver, besides which I had the spectrum sensitiveness of other salts of silver by me, and I knew the air thickness and the spectral luminosity on the day when these measures were taken. Assuming that the ultra violet rays obeyed the same law as the visible rays, it was easy to calculate the photographic value of each ray for any air thickness; and this I accordingly did.

Using a value of ·00183 for k in Lord Rayleigh's formula, the same process as before was gone through, supposing a plate of bromo-iodide of silver had been used: the areas were for 0, 1, 2, 3, and 4 atmospheres, 835, 621, 457, 340, and 254. The values arrived at by the logarithmic formula were 837, 621, 461, 342, and 254. This gave $\mu=\cdot2980$, and the coefficient of transmission, ·742.

Taking chloride of silver as the sensitive salt, its spectrum value being known, and proceeding as before, the following areas were found for 0, 1, 2, 3, and 4 atmospheres: 693, 413, 246, 147, and 87. Calculated by the logarithmic formula, they became 689, 413, 248, 148, and 87, $\mu=\cdot5115$, and the coefficient of transmission $=\cdot603$.

Taking these two results it is evident that before any definite knowledge of a value of a star magnitude by photography can be arrived at (supposing all stars were of the same colour) a definite acquaintance must be made with the kind of sensitive salt that is employed. Thus the bromo-iodide plate gave a value of transmission of ·742; the chloride, of ·603; whilst the optical value was only ·880 when $k = 00183$.

Now it may be considered that these results are only theoretical deductions, but luckily they are capable of proof in the laboratory by experiment.

Water can easily be made turbid by alcoholic solutions of mastic or by other fine precipitates, and last year General Festing and myself proved that these artificially prepared turbid media strictly obeyed Lord Rayleigh's law. Now it was easy, having proved this, to ascertain whether when the light passing through such media was measured integrally by optical and photographic means the logarithmic formula was followed.

Experiment showed that it did so. Here is one of several results. A cell measuring exactly 6 inches by 4 inches was taken, and the values of light passing through these thicknesses of clear and turbid water were found to be—

	4-in.	6-in.
Clear	75·6	75·4
Turbid	26·5	15·5

Taking the clear or incident light as the mean of the two, 75·5, and using the 6 inches of the turbid medium as lying on the curve, the logarithmic formula gave $\mu = \cdot 2639$ in the formula

$$I' = I\epsilon^{-\mu x},$$

and the absorption consequently for 4 inches of the fluid $= 26 \cdot 3$.

Taking the photographic values a bromo-iodide plate was exposed to the clear, the 4-inch and the 6-inch turbid water for definite times, and the values of the light acting on the plate found to be

4-inch clear	1
1 – 4-inch turbid	$\frac{1}{10 \cdot 125}$
6-inch turbid	$\frac{1}{33 \cdot 1}$

Using the logarithmic formula as before, taking the clear and 6-inch turbid light as lying on the curve, the value of the 4-inch turbid would be $\frac{1}{10 \cdot 3}$, and the value of $\mu = \cdot 5832$.

The optical value of μ is 2·639. The ratio of the optical to the photographic value of μ is 1 : 2·21, as determined by experiment.

Taking the optical and photographic values of μ, say, for a value of $k = 001183$, they are ·1324 and 2980, or 1 : 2·25.

This shows, I think, that the values of μ determined experimentally in the laboratory and by appeal to the atmosphere are as consistent as can be expected.

Further we have this curious fact, that if we take the optical value of unanalysed light passing through the atmosphere, and also the photographic value of the same, we are able to form two equations which give the value of k, which must be applied to every wave-length, and also enable a curve of light to be constructed which takes into account general absorption by mist which is not composed of sufficiently fine particles to scatter light. Calculation shows that k is derived from either by dividing μ by 110 and 250 respectively when k is less than ·0013. These gradually alter in value till they become 104 and 255, when $k = \cdot 0015$ is reached. The values of 110 and 255 are inverse fourth power of wave-lengths in the green and violet respectively, showing that when light is integrally measured it is equivalent to measuring definite rays in these parts of the spectrum.

I have not touched upon the question of how star magnitudes are to be measured, but merely ventured to give a means of applying a correction for loss caused by the atmospheric transmission. It will be seen how much greater is the loss of those rays which are photographically active than the visual rays, and it would appear that to enable a proper correction to be

made to photographs of stars taken at different times, the visual absorption at the time should also be calculated. I am not alluding now to the fact that the angle included in any star photograph is small; stars on such a plate may be compared *inter se*; but in order to make comparisons general the atmospheric absorption for the photographically active rays must be studied, and it is absolutely necessary that the spectrum value of the sensitive salt should be known beforehand.

Photographic Search for the Minor Planet Sappho.
By Isaac Roberts.

From 1872 to the present time no observations of the minor planet *Sappho* have been published except those of 1882, in which year Mr. Gill took measures for a determination of the solar parallax.

Mr. Bryant, who is engaged in determining the orbit of this planet, prepared an ephemeris for the opposition of this year, and in order that no time might be lost in identifying it from the neighbouring faint stars, and further that the error of the ephemeris might be obtained as early as possible, he sent, on December 16, 1886, the positions he had calculated, and appealed to me to find the planet if possible by photography.

The brightness of the planet is estimated at eleventh magnitude, and since its orbital movement in sixty minutes is equal to about 4·2 times its photographic diameter, the trail it would leave would probably not exceed in density a thirteenth magnitude star.

On December 30 I obtained, with an exposure of sixty minutes between sidereal time at Maghull, $7^h\ 35^m$ and $8^h\ 35^m$, a negative of which the accompanying photograph marked chart No. 1 is an enlargement to three diameters. The trail of the planet is to be seen near the centre, and to make it more easily recognisable a white circle is drawn round it.

Another photograph was taken on January 1, and a third on the 14th. These are marked respectively charts Nos. 2 and 3. Each of the three charts shows the difficulties to be encountered in finding a planet so faint as this one amongst such numbers of other faint stars, and as an illustration I may refer to chart No. 3 upon which I estimate that there are about 2,000 stars below the seventh magnitude on each square degree.

Mr. Bryant informs me that the error of the ephemeris deduced from the photographs is in very close agreement with that from two meridian observations of the planet made about the same time at Dunecht, results that must be considered satisfactory.

This is probably the first instance in which photography has been successfully applied to the purpose here described, and as an historical fact it may be worth recording. There are also